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Summary

The research project had two components.

In the first part, we developed a numerical method, based on the wavelet transform, for the solution of partial differential equations. Singularities and sharp transitions in solutions of partial differential equations model important physical phenomena, which are hard to simulate with conventional numerical methods. In collaboration with Pfr. Papanicolaou and Bacry, we introduced a numerical scheme based on the orthogonal wavelet transform, that adapts the computational resolution in space and time to the regularity of the solution. This scheme saves computations by concentrating the computational effort in regions where singularities or sharp transitions occur. It has been tested on the Burgers equation.

The second part of the project concerns the detection, characterization and processing of singularities in signals. In collaboration with Hwang, we developed theorems and algorithms to detect and characterize singularities in complex signals, by tracking the local maxima of the wavelet transform. Applications to the removal of noises and the characterization of fractals have been developed. The reconstruction of functions from the local maxima of the wavelet transform was studied with Zhong. An algorithm that reconstructs one and two-dimensional signals from the wavelet transform maxima was derived. This algorithm reconstructs images from their multiscale edges, which verifies a conjecture made 15 years ago by David Marr. A compact image coding algorithm was derived.

A documented wavelet transform software that processes one-dimensional signals and images has been distributed to research laboratories as well as industries. With this software, we taught a course on the wavelet transform at Martin Marietta, for engineers coming from several companies involved in the defense industry.

Wavelet Local Extrema

Final report to AFOSR Grant: AFOSR-90-0040 (11/1/89 - 10/31/92)

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1. Overview

The research project had two components. In the first part, we developed a numerical method, based on the wavelet transform, for the solution of partial differential equations. Singularities and sharp transitions in solutions of partial differential equations model important physical phenomena, which are hard to simulate with conventional numerical methods. In collaboration with Pfr. Papanicolaou and Bacry, we introduced a numerical scheme based on the orthogonal wavelet transform, that adapts the computational resolution in space and time to the regularity of the solution. This scheme saves computations by concentrating the computational effort in regions where singularities or sharp transitions occur. It has been tested on the Burgers equation. The results of this work are summarized in Section 2.

The second part of the project concerns the detection, characterization and processing of singularities in signals. In collaboration with Hwang, we developed theorems and algorithms to detect and characterize singularities in complex signals, by tracking the local maxima of the wavelet transform. Applications to the removal of noises and the characterization of fractals have been developed. The reconstruction of functions from the local maxima of the wavelet transform was studied with Zhong. An algorithm that reconstructs one and two-dimensional signals from the wavelet transform maxima was derived. This algorithm reconstructs images from their multiscale edges, which verifies a conjecture made 15 years ago by David Marr. A compact image coding algorithm was derived. The results are presented in Section 3.

In parallel to our research, we made an effort to transfer our knowledge to industries that are potentially interested by the new technics that we developed. A documented wavelet transform software that processes one-dimensional signals and images has been distributed to research laboratories as well as industries. With this software, we taught a course on the wavelet transform at Martin Marietta, for engineers coming from several companies involved in the defense industry. Lecture notes of more than 100 pages were written for the participants of the course.

2. Wavelet Space-Time Adaptive Method for Partial Differential Equations

This project was done in cooperation with Pfr. George Papanicolaou and a doctoral student: Emmanuel Bacry. Singularities and sharp transitions in solutions of partial differential equations model important physical phenomena such as beam focusing in nonlinear optics, the formation of shock waves in compressible gas flow, the formation of vortex sheets in high Reynolds number incompressible flows, etc. A characteristic feature of such phenomena is that the complex behavior occurs in a small region of space and intermittently in time. This makes them particularly hard to simulate numerically by solving the partial differential equations with conventional numerical methods, prompting the development of adaptive numerical methods. In these methods most of the computational effort is concentrated near regions where singularities or sharp transitions occur. We have studied a numerical method for solving partial differential equations based on the wavelet transform, which is adaptive both in space and time [4].

The multiresolution structure of wavelet orthonormal bases is a simple and effective framework for spatial adaptive algorithms. Instead of refining the computations through nested grids of successively finer meshes, as in classical adaptive grid algorithms, wavelet orthonormal bases implement adaptive refinement by successively adding layers of "details" that increase the resolution of the approximation locally. Communication between the different layers of details is regulated automatically by the orthogonality of the basis functions. The order of approximation of this spatial discretization depends upon the wavelet that is used.

For many evolution problems that are solved numerically with a space adaptive scheme, it is necessary to adapt the time discretization to the spatial resolution. If we use a time step Δt , it must be adapted to the highest resolution that is encountered over the whole spatial domain, even if this high resolution is maintained over a very small domain. If the spatial resolution is refined locally, the time step Δt must also be refined to maintain the stability and accuracy of the numerical scheme. This means that a local spatial refinement, even over a small domain, increases the global numerical complexity quite substantially. For adaptive numerical methods based on wavelets, it is thus important to have a local time discretization. We have introduced a new algorithm that adapts the time discretization to the resolution parameter that appears in a wavelet orthonormal basis [4]. The stability of the algorithm has been demonstrated numerically for the heat equation as well as the linear advection equation and the Burgers equation. We studied the trade-off between the gain in numerical complexity and the precision of the solution, for the one-dimensional Burgers equation [4].

3. Singularity Detection, Characterization and Processing

Singularities and irregular structures often carry the most important information in signals. In images, the discontinuities of the intensity provide the locations of the object contours, which are particularly meaningful for recognition purposes. For many other types of signals, from electro-cardiograms to radar signals, the interesting information is given by transient phenomena such as peaks. In physics, it is also important to study irregular structures to infer properties about the underlined physical phenomena.

We studied the detection of the signal singularities from the local maxima of the wavelet transform modulus. A mathematical analysis of the modulus maxima properties has been done in collaboration with Wen Liang Hwang. A theorem proves that the local maxima of a wavelet transform allows us to detect all the singularities of a function [3]. In mathematics, the local regularity of a function is often measured with Lipschitz exponents. We showed that these local Lipschitz exponents can be measured from the evolution across scales of the values of the wavelet transform, at the maxima positions [3]. We derived practical algorithms to analyze isolated or non-isolated singularities in signals. The wavelet transform has a particular behavior when singularities have fast oscillations. We showed that the local frequency of the oscillations can be measured from the points where the modulus of the wavelet transform is locally maximum both along the scale and the spatial variable [3].

An important issue is to understand how much information is carried by the local maxima of a wavelet transform modulus. Is it possible to reconstruct the original signal or a close approximation from these modulus maxima? Meyer proved that the local maxima of a wavelet transform modulus do not characterize uniquely a function. However, a numerical algorithm developed in collaboration with Sifen Zhong [2] is able to reconstruct a close approximation of the original signal. A proof of the convergence of the algorithm has been derived [2]. We can thus process the singularities of a signal by modifying the local maxima of its wavelet transform modulus and then reconstruct the corresponding function. In images, the wavelet transform local maxima correspond to edges detected at different scales. The reconstruction algorithm recovers a close approximation of the original image from the multiscale edges. The differences between the original image and the reconstructed one are not visible. This confirms the conjecture of David Marr that was made 15 years ago. This algorithm allows us to process the singularities of a signal by modifying the local maxima of its wavelet transform modulus, and then reconstruct the corresponding function. We studied applications to noise removal and compact

image coding.

To discriminate a noise from a signal requires some a priori information on the structural properties of the noise and the signal. In image processing, we often have some prior information on the noise and the signal singularities. For example, a Gaussian white noise has singularities whose Lipschitz exponents are smaller than the exponents of most singularities encountered in images. Moreover, the image singularities often belong to smooth edge curves that are the boundaries of regular image structures. On the contrary, a white noise does not create such edge curves. The noise removal algorithm developed in collaboration with Wen Liang Hwang [3], takes advantage of this prior information in order to remove part of the noise from the multiscale edge representation. The algorithm analyzes the multiscale edges of the noisy signal and discriminates the singularities created by the noise from the singularities of the signal. The edge points of the noise are removed and we also regularize the singularities of the signal that are affected by the noise. A cleaned signal is then reconstructed from the multiscale edge representation. This procedure suppresses the singularities created by the noise but does not smooth globally the signal. Hence, the sharp signal variations are not degraded.

An important problem in image processing is to code images with a minimum number of bits for transmission or storage. To obtain high compression rates in image coding, we cannot afford to code all the information available in the image. It is necessary to remove part of the image components that are not important for the visualization. Since edges provide meaningful features for image interpretation, it is natural to represent the image information with an edge based representation, in order to select the information to be coded. The coding algorithm developed in collaboration with Sifen Zhong [2], involves two steps. First we select the edge points that we consider important for the visual image quality. This preprocessing is identical to the feature extraction stage of a pattern recognition algorithm. We then make an efficient coding of this edge information. We obtain a compression of the original image data by factors over 30 but some of the finer image details are not encoded. The image recovered from this code is sharp and does not have distortion such as Gibbs phenomena.

4. Publications of Research Supported by the Grant

Ph.D. Theses

Sifen Zhong "Edge Representation from Wavelet Transform Maxima", NYU, Sept. 1990.

Journal Articles

1. S. Mallat, "Zero-Crossings of a Wavelet Transform", *IEEE Transactions on Information Theory*, vol. 37, p. 1019-1033, July 1991.
2. S. Mallat, S. Zhong, "Characterization of signals from multiscale edges", To appear in *IEEE Transaction on Pattern Analysis and Machine Intelligence*.
3. S. Mallat, W.L. Hwang, "Singularity detection and processing with wavelets", *IEEE Transactions on Information Theory*, March 1992.
4. E. Bacry, S. Mallat, G. Papanicolaou, "A wavelet based space-time adaptive numerical method for partial differential equations", *Mathematical Modeling and Numerical Analysis*, December 1992.

Book Chapters

5. S. Mallat, S. Zhong, "Wavelet transform and multiscale edges", *Wavelet and Applications* ed. Coifman et. al., Jones and Bartlett, 1991.
6. J. Froment and S. Mallat, "Second generation compact image coding with wavelets", in *Wavelets-A Tutorial in Theory and Applications*, ed. C. Chui, p. 655-678, Academic Press, January 1992.

Patent Disclosure

7. S. Mallat and Z. Zhang, "Method and apparatus for encoding and decoding signals with structures", *US patent disclosure*, March 11 1992.

Proceedings of Conferences

8. S. Mallat, S. Zhong "Signal characterization from multiscale edges" *Proc. of 10th International Conference on Pattern Recognition* Atlantic City, June 1990.
9. S. Zhong, S. Mallat "Compact image representation from multiscale edges" *Proc. of 3rd. International Conference on Computer Vision* Osaka, December 1990.
10. S. Mallat, S. Zhong "Compact image coding from edges with wavelets", *Proc. of International Conference in Acoustics Speech and Signal Processing* Toronto, May 1991.
11. S. Mallat, W.L. Hwang, "Characterization of singularities", *Proc. of NATO Advanced Study Institute on Probabilistic and Stochastic Methods in Analysis*, El Ciocco, Italy, July 1991.
12. A. Chambolle, D. Geiger, S. Mallat, "Un algorithme multi-echelle de mise en correspondance stereo base sur les champs markoviens," in *Proc. of 13th GRETSI Conf. on Signal and Image Processing*, Juan-les-Pins, France, September 1991.
13. W.L. Hwang, S. Mallat, "Singularities and noise discrimination with wavelets", *Proc. of International Conference in Acoustics Speech and Signal Processing* San Francisco, March 1992.
14. J. Froment, S. Mallat, "Compact image coding with the wavelet maxima", *International conference on Wavelets and Applications*, Toulouse, France, June 1992.
15. E. Bacry, S. Mallat, G. Papanicolaou, "Time and space adaptive scheme for non-linear PDE", *International conference on Wavelets and Applications*, Toulouse, France, June 1992.
16. E. Bacry, S. Mallat, G. Papanicolaou, "Wavelet based numerical scheme for PDE", *International Conference on Spectral and High Order Methods*, Montpellier, France, June 1992.

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